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Inaccurate Experiments and Environment-Induced Superselection Rules

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It is argued that approximate superselection rules induced by the environment cannot account for the emergence of definite measurement results in single experiments. The reason for this is that the inaccuracy necessary for an experiment failing to distinguish between exact and approximate mixtures requires the pointer observable to be strictly classical. This is shown in the case that the observables of physical systems generate W^*-algebras.

1. INTRODUCTION

In the last two decades there has been a lively discussion in the literature on whether or not the pointer observable in a quantum measurement should be classical.

The initial argument for the use of a classical pointer observable is due to Hepp (1972): If the pointer observable is classical, then apparatus states corresponding to different pointer values are disjoint. A linear superposition of disjoint states is always equivalent to an incorherent mixture of such states. In a statistical interpretation, Lüders' rule describes how the pure superposed state of the apparatus is reduced to a mixture. If the pointer observable is classical, this is not necessary: the linear superposition and the incoherent mixture of apparatus states with different pointer values are anyway equivalent.

In an individual interpretation, this is not yet a satisfactory explanation for why we get a definite measurement result. The mixture of apparatus states with different pointer values in general does not admit an ignorance interpretation, because the decomposition into pure states is not

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unique. However, if the pointer observable is classical, then the decomposition of the mixed apparatus states into pure states is unique. This was shown for a pointer observable with discrete spectrum in Landsman (1991, Section 4.5) and Busch *et al.* (1991), Theorem 5.2.1.

So it seems that two major obstacles to a solution of the measurement problem disappear if we have a classical pointer observable. But two other problems crop up: First, it has to be explained how classical observables arise in a (possibly infinite) quantum system. I will not deal with this problem here.

Second, an automorphic time evolution of the joint system can in no finite time lead to the disjoint states corresponding to different pointer values. In the infinite-time limit, convergence to disjoint states can be achieved in the weak*-topology (Hepp, 1972), but not in the uniform topology (Bell, 1975). Since one has to explain the occurrence of definite measurement results in a finite time, one has probably to accept that the time evolution of the joint system cannot be automorphic if the pointer observable is classical. This seems reasonable if the joint system is open.

There have been several suggestions (for example, Zurek, 1982; Dieks, 1989; Hannabus, 1984) that—for all practical purposes—the occurrence of definite results in single experiments can be explained by the influence of the environment *alone*, without assuming the pointer observable to be strictly classical. It is argued in Zurek (1982) that the environment induces a superselection rule in the following sense. The coupling of the apparatus to an environment reduces the initial superposition of eigenstates of the pointer observable quickly into a state which is almost a mixture. The coherence present in the superposed state is dislocalized into the many degrees of freedom of the environment. Thus the final state of the joint system is *approximately* diagonal in the pointer basis. In the modal interpretation of Dieks (1989), this final state leads to a definite measurement result in every single experiment. Our experiments of finite accuracy cannot distinguish between a true mixture (with vanishing terms off the diagonal) and a pure final state (with very small terms off the diagonal) which is approximately a mixture. Thus the correlations are unobservable (in inaccurate experiments), which is interpreted as having an approximate, environment-induced superselection rule. The superselection rule is not a strict one because there is not necessarily a superselection operator, i.e., a nontrivial operator commuting with all observables.

In this paper I will argue against such a solution of the problem. The main line of the argument is: Only inaccurate experiments fail to distinguish between the true mixture and the approximate mixture. Generalizing a proof of Breuer (1992), I show that inaccurate experiments require the pointer observable to be strictly classical. Therefore the influence of the environment cannot explain how quantum measurements work as long as it leads only to approximate superselection rules.

2. INACCURATE EXPERIMENTS

In this section I describe two ways in which experiments can be inaccurate and then abstract from the examples a notion of inaccuracy which I think fits most realistic experiments.

First example: Consider the case where a digital point is used in a measurement of an observable A with continuous spectrum. After the experiment we register one number k as result, but say that the actual value of A might be in the interval $[k-\epsilon, k+\epsilon]$. There is a state ψ_k in which A really has the value k. But all the states in which A has a value in $[k-\epsilon, k+\epsilon]$ might as well have led us to register the result k. So there is an ϵ-neighborhood of ψ_k leading to the same pointer reading.

It may seem that this kind of inaccuracy is excluded if the value k is isolated by more than ϵ from the other possible results, as, for example, in the measurement of the spin of a particle in a given direction z. Still—and this is my *second example*—a Stern–Gerlach experiment can be inaccurate in the following way: The counter registers particles leaving a magnetic field whose direction might deviate by a small angle from z. Therefore these particles give rise to the same pointer reading as particles with spin exactly in z. Such a kind of inaccuracy was described in Primas (1990) by a *finite partition of the Hilbert space* on which the measured observable operates.

Common to the two examples is the following notion of inaccuracy. There are some "typical" final states ψ_k in which the quantity we want to measure really has the value k. After an inaccurate experiment, however, all states in a sufficiently small ϵ-neighborhood of some ψ_k give rise to the same pointer reading as ψ_k.

Let us try to formulate in a more general framework this notion of inaccuracy. Assume that the observables of the measured system and of the apparatus generate W^*-algebras $\mathscr{A}_{\mathscr{S}}$, $\mathscr{A}_{\mathscr{M}}$, respectively. Let ρ_k be the pure state of the joint system after an experiment which leaves the system in the state $\rho_k|_S$. (The states $\rho_k|_S$ are what was called ψ_k above.) One might be tempted to characterize the above inaccuracy by saying that there is a nonempty family $\{\rho_k\}_{k\in K}$ of pure states of the joint system for which

$$(\exists\epsilon)(\forall\rho): \quad \|\rho|_s - \rho_k|_s\| < \epsilon \quad \text{for some} \quad k \Rightarrow \rho(P) = \rho_k(P)$$

(Take $\epsilon < \inf_{i\neq k} \|\rho_k - \rho_i\|/2$ to avoid overlapping of the ϵ-neighborhoods of the ρ_k.) This requirement would mean that if the norm-distance of $\rho|_S$

to $\rho_k|_S$ is small enough, then the corresponding pointer readings should be the same.

But states ρ of the joint system whose restriction $\rho|_S$ to the observed system is close to $\rho_k|_S$ can be as far from ρ_k as states can be. So if we expected all such states to yield the same expectation value for the pointer obsevable, then the pointer observable would have to be constant. We can require something like the above only for states *after* the experiment.

The measurement interaction couples the system states to certain apparatus states. So not all states ρ with $\|\rho|_s - \rho_k|_s\| < \epsilon$ are possible states of the joint system after the experiment. Let us assume that the coupling established by the measurement interaction is continuous:

$$(\forall\delta)(\exists\epsilon)(\forall\rho\in(\mathscr{A}_{\mathscr{S}}\bar{\otimes}\mathscr{A}_{\mathscr{M}})^*):\quad \|\rho|_s - \rho_k|_s\| < \epsilon \Rightarrow \|\rho - \rho_k\| < \delta$$

States ρ satisfying $\|\rho|_s - \rho_k|_s\| < \epsilon$ but not $\|\rho - \rho_k\| < \delta$ do not have the relation between system and apparatus necessary for the inference of information about the system from information about the apparatus. Therefore they are not possible states after the experiment. This leads to the following requirement for an experiment to be inaccurate: There is an ϵ such that for all pure states ρ of the joint system *after the experiment* we have $\|\rho|_s - \rho_k|_s\| < \epsilon \Rightarrow \rho(P) = \rho_k(P)$. This definition of finite measurement accuracy in conjunction with continuity of coupling is equivalent to the following requirement.

Finite measurement accuracy: There exists a nonempty family $\{\rho_k\}_{k\in K}$ of pure states of the joint system (the "typical" final states) and a δ, $0 < \delta < \inf_{i\neq k}\|\rho_k - \rho_i\|/2$ such that the pointer observable $P\in\mathscr{A}_{\mathscr{S}}\bar{\otimes}\mathscr{A}_{\mathscr{M}}$ has the same expectation value in all pure states ρ of the joint system satisfying $\|\rho - \rho_k\| < \delta$ for some k.

3. THE MAIN RESULT

(A) Assuming that for all $Z\in\mathscr{Z}(\mathscr{A}_{\mathscr{S}}\bar{\otimes}\mathscr{A}_{\mathscr{M}})$ there is a ρ_k with $\rho_k(Z) \neq 0$, an experiment can be of finite accuracy if and only if the pointer observable P is classical.

(B) An experiment with pointer observable P can be of finite accuracy if and only if there is a classical observable $\bar{P}$ with $\rho_k(P) = \rho_k(\bar{P})$, $\forall k\in K$. (Since the typical final states ρ_k cannot distinguish between P and $\bar{P}$, P and $\bar{P}$ give rise to the same pointer reading.)

Proof of A: Denote by $\pi_{\rho_k}(\mathscr{A}_{\mathscr{S}}\bar{\otimes}\mathscr{A}_{\mathscr{M}})$ the GNS representation with respect to the state ρ_k of $\mathscr{A}_{\mathscr{S}}\bar{\otimes}\mathscr{A}_{\mathscr{M}}$ on the Hilbert space $\mathscr{H}_{\rho_k}$, where there is a cyclic vector Ω_{ρ_k} such that $\rho_k(A) = \langle\Omega_{\rho_k}|\pi_{\rho_k}(A)|\Omega_{\rho_k}\rangle$ for all $A\in\mathscr{A}_{\mathscr{S}}\bar{\otimes}\mathscr{A}_{\mathscr{M}}$.

Step 1. $\bigcap_{k\in K} \ker \pi_{\rho_k} = \{0\}$. Take $A \in \bigcap_{k\in K} \ker \pi_{\rho_k}$. First of all we observe that A cannot be in $\mathscr{Z}(\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}})$: If A were in $\mathscr{Z}(\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}})$, then there would be a ρ_k with $\rho_k(A) \neq 0$.

Now, $\bigcap_{k\in K} \ker \pi_{\rho_k}$ is a weakly closed two-sided ideal of $\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}}$. It follows (see, e.g., Bratteli and Robinson, 1987, Theorem 2.4.22) that there is a projection $Q \in \mathscr{Z}(\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}})$ such that $\bigcap_{k\in K} \ker \pi_{\rho_k} = Q(\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}})Q$. So $\bigcap_{k\in K} \ker \pi_{\rho_k} \neq \{0\}$ would imply that $\bigcap_{k\in K} \ker \pi_{\rho_k} \cap \mathscr{Z}(\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}}) \neq \{0\}$. But since there is no central element in $\bigcap_{k\in K} \ker \pi_{\rho_k}$, $\bigcap_{k\in K} \ker \pi_{\rho_k} = \{0\}$.

Step 2. $P \notin \mathscr{Z}(\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}})$ implies

$$(\forall\delta)(\exists k \in K)(\exists \rho \text{ pure}): \quad \|\rho - \rho_k\| < \delta, \quad \rho(P) \neq \rho_k(P).$$

$P \notin \mathscr{Z}(\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}})$ implies that there exists a $B \in \mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}}$ such that $[P, B] \neq 0$. From the first step it follows that there is a k with $\pi_{\rho_k}([P, B]) \neq 0$. So $\pi_{\rho_k}(P) \notin \mathscr{Z}(\pi_{\rho_k}(\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}}))$. Since ρ_k is pure, π_{ρ_k} is irreducible, so $\pi_{\rho_k}(P)$ cannot be constant. Take any vector state $\psi \in \mathscr{H}_{\rho_k}$ with

$$\langle \psi | \pi_{\rho_k}(P) | \psi \rangle \neq \langle \Omega_{\rho_k} | \pi_{\rho_k}(P) | \Omega_{\rho_k} \rangle = \rho_k(P)$$

Define $\psi_\alpha := \alpha\Omega_{\rho_k} + (1-\alpha)\psi$ for $0 < \alpha < 1$. Take $\rho_\alpha(A) := \langle \psi_\alpha | \pi_{\rho_k}(A) | \psi_\alpha \rangle$ for all $A \in \mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}}$. Now it can be checked that all ρ_α are pure states satisfying $\rho_\alpha(P) \neq \rho_k(P)$. If we choose α close enough to one, then $\|\rho_\alpha - \rho_k\| < \delta$.

Step 3. $P \in \mathscr{Z}(\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}})$ implies

$$(\exists\delta)(\forall k \in K)(\forall \rho \text{ pure}): \quad \|\rho - \rho_k\| \in \delta \Rightarrow \rho(P) = \rho_k(P)$$

Take $\delta < \inf_{i \neq k} \|\rho_i - \rho_k\|/2 < 2$ and let ρ be a pure state with $\|\rho - \rho_k\| \delta < 2$ for some k. A theorem by Glimm and Kadison (1960) shows that in this case the representations π_ρ and π_{ρ_k} are unitarily equivalent. All classical observables have the same value in the states ρ and ρ_k, so $\rho(P) = \rho_k(P)$.

Proof of B. The family $\{\rho_k\}_{k\in K}$ defines a two-sided ideal $\mathscr{I} \subset \mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}}$ by

$$\mathscr{I} := \{A |_{\rho_K}(B^*AC) = 0, \forall B, C \in \mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}}, \forall k \in K\} = \bigcap_{k\in K} \ker \pi_{\rho_k}$$

There exists a projection $Q \in \mathscr{Z}(\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}})$ such that $\mathscr{I} = Q(\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}})Q$. Now, $\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}}$ can be written as

$$\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}} = (\mathbf{1} - Q)(\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}})(\mathbf{1} - Q) \oplus Q(\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}})Q =: \mathscr{A}' \oplus \mathscr{A}''$$

Since for every $Z' \in \mathscr{Z}(\mathscr{A}')$ there is a ρ_k with $\rho_k(Z') \neq 0$, we have

$\bigcap_{k\in K} \ker \pi_{\rho_k}(\mathscr{A}' + 0) = \{0\}$. Every observable $A \in \mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}}$ can be written as $A = A' + A''$, every state ρ on $\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}}$ as $\rho = \rho' + \rho''$.

If there is a $\bar{P} \in \mathscr{Z}(\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}})$ with $(\forall k \in K)$: $\rho_k(P) = \rho_k(\bar{P})$, then $\bar{P} = \bar{P}' + \bar{P}''$, $P = \bar{P}' + P''$. From the third step of the proof of (A) it follows that

$$(\exists \delta)(\forall k \in K)(\forall \rho' \text{ pure states on } \mathscr{A}'): \quad \|\rho' - \rho'_k\| < \delta \Rightarrow \rho'_k(\bar{P}') = \rho'(\bar{P}')$$

Since $\|\rho' - \rho'_k\| < \|\rho - \rho_k\|$, all states ρ with $\|\rho - \rho_k\| < \delta$ satisfy $\rho(\bar{P}' + 0) = \rho'(\bar{P}') = \rho'_k(\bar{P}') = \rho_k(\bar{P}' + 0)$. So the requirement of finite accuracy is satisfied for the pointer observable $\bar{P}' + 0$.

If there is no $\bar{P} \in \mathscr{Z}(\mathscr{A}_{\mathscr{S}} \bar{\otimes} \mathscr{A}_{\mathscr{M}})$ with $(\forall k \in K)$: $\rho_k(P) = \rho_k(\bar{P})$, then $P' \notin \mathscr{Z}(\mathscr{A}')$. From the second step of the proof of (A) it follows that $(\forall \delta)(\exists k \in K)(\exists \rho'$ pure states on $\mathscr{A}')$: $\|\rho' - \rho'_k\| < \delta$ but $\rho'_k(P') \neq \rho'(P')$. Taking $\rho := \rho' + \rho''_k$, it follows that $\|\rho - \rho_k\| < \delta$, but $\rho(P) \neq \rho_k(P)$.

4. CONCLUSION

The main aim of this paper was to show that—contrary to suggestions in Zurek (1982) and Dieks (1989)—approximate superselection rules induced by the environment cannot explain why in single experiments we have a definite pointer reading. This can only be concluded if experiments are inaccurate, which in turn requires pointer observables to be strictly classical.

But this does not mean that the environment does not have any role to play. On the contrary: it might explain why the joint system follows the nonautomorphic time evolution necessary to arrive in finite time at disjoint final states. Furthermore, the coupling to an infinite-dimensional environment might provide a mechanism for the emergence of classical observables in an apparatus described by quantum theory. These interesting questions have not been dealt with in this paper.

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